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"Group Actions and Homogeneous Spaces"
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ABSTRACTS OF LECTURES

Comenius University
Faculty of Mathematics, Physics, and Informatics
Bratislava, Slovakia

On the first homology of automorphism groups of G-manifolds with codimenison one orbit

Kōjun Abe (Shinshu Univ.)

We have been calculated the first homology group of the equivarinat diffeomorphism, the equivariant Lipschitz homeomorphism group and the equivariant homeomorphism group of G-manifolds. Those first homology groups are quite different by the category of the automorphism group. We shall describe the results and the outline of the proof.

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REAL BOTT MANIFOLDS AND ACYCLIC DIGRAPHS

SUYOUNG CHOI

Abstract

A *Small cover*, defined by Davis and Januszkiewicz [2], is an n-dimensional closed smooth manifold M with a smooth action of standard real torus $\mathbb{Z}_2^n$ action such that the action is locally isomorphic to a standard action of $\mathbb{Z}_2^n$ on $\mathbb{R}^n$ and the orbit space $M/\mathbb{Z}_2^n$ can be identified with a simple (combinatorial) polytope. In this talk, we assume that the orbit polytope is a hypercube. Small covers over cubes, called the real *Bott manifolds*, are obtained as iterated $\mathbb{R}P^1$ bundles starting with a point, where each stage is the projectivization of a Whitney sum of two real line bundles. In this talk, we discuss about the topological classification of real Bott manifolds. In fact, by Kamishima and Masuda [3], two real Bott manifolds are diffeomorphic if their cohomology rings with $\mathbb{Z}_2$ coefficients are isomorphic. In addition, Masuda [4] has explained the diffeomorphism classes of them by 3 operations of the set of real Bott manifolds. In other words, two Bott manifolds are diffeomorphic if one is transformed to the other through a sequence of the three operations.

On the other hand, Choi [1] has found a 1-1 correspondence between the set of real Bott manifolds and the set of combinatorial objects, acyclic digraphs. So we may regard operations of real Bott manifolds as operations of digraphs. One of operations is well-known for the *local complementation*. We investigate many invariants under the these operations. This work is jointly with Sangil Oum.

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Symplectomorphisms, diffeomorphisms and circle actions on symplectic manifolds

Bogusław Hajduk

University of Wrocław and University of Warmia and Mazury

In this talk I will discuss relations between smooth and symplectic objects on closed manifolds. Basic problems in this topic include: existence of symplectic forms, relations between symplectomorphisms and diffeomorphisms, existence of symplectic circle actions. A lot is known on such questions in dimension four, but in dimensions ≥ 6 they are wide open.

I will explain relations between the following questions:

• If a symplectic manifold is homeomorphic to the torus T^{2n}, is it diffeomorphic to T^{2n}?

• When a diffeomorphism of T^{2n} is isotopic to a symplectomorphism?

$\star$ Does $M \times S^1$ admit a symplectic structure only if M fibers over S^1?

$\star$ Does any closed symplectic manifold with a (smooth) circle action admit a symplectic circle action?

I will give examples to show that answers to these questions might be not simple in higher dimensions. In particular there are diffeomorphisms of T^{2n} supported in a disc which are not isotopic to symplectomorphisms and circle actions on symplectic manifolds which are not equivalent to symplectic actions.

Large part of my talk is a report on a joint project with Aleksy Tralle and Rafał Walczak.

On the first Pontrjagin class of homotopy complex projective spaces

Yasuhiko KITADA

(Yokohama National University)

Let M^{2n} be a closed smooth manifold homotopy equivalent to the complex projective space $\mathbb{C}\mathrm{P}(n)$. Pontrjagin classes have been studied to classify homotopy complex projective spaces. It is known that the first Pontrjagin class $p_1(M)$ has the form

$$p_1(M) = ((n+1) + 24\gamma(M))x^2$$

for some integer $\gamma(M) \in \mathbb{Z}$, where $x \in H^2(M;\mathbb{Z})$ is the generator. When n is odd, many facts are known; if there exists a framed $(n+1)$-manifold with Kervaire invariant one, then $\gamma(M)$ can take any integer. In other dimensions, $\gamma(M)$ can take any even number. Conversely, $\gamma(M)$ is even at least when $n \equiv 1 \mod 4$. However it is hard to get meaningful results when n is even.

Let $\nu_2(n)$ denote the 2-order of n, i.e. the exponent of the prime factor 2 in the prime decomposition of n. We have been doing calculations of the surgery index obstruction of normal maps of $\mathbb{C}\mathrm{P}(n)$ when n is even. The calculation gets harder as $\nu_2(n)$ increases. We have now succeeded in proving the following theorem:

Theorem. If n is even and $\nu_2(n) < 6$, then $\gamma(M)$ is even.

The restriction on the 2-order of n is not best possible. By direct calculation, we know that the theorem is also true for $n = 64$, but we do not know the general proof for other dimensions with larger 2-orders. We hope that we can remove the restriction on the 2-order of n in the future.

Let $n = 2k$ and $f : M^{4k} \to \mathbb{C}\mathrm{P}(2k)$ be a homotopy equivalence and $\varphi : \mathbb{C}\mathrm{P}(2k) \to F/O$ be its normal map. Since $\gamma(M)$ is even, we can conclude that the pullback of the two dimensional Kervaire class $\varphi^*(K_2)$ vanishes. Then we know that other pullbacks $\varphi^*(K_{2^{j+1}-2})$'s simultaneously vanish in dimensions less than $2k$. Let us consider the case where the normal map φ has an extension $\mathbb{C}\mathrm{P}(n+1) \to F/O$. Unless $k+1$ is not a power of 2, by the simultaneous vanishing of the Kervaire classes, using Sullivan's formula, we conclude that the $(4k+2)$ dimensional Kervaire surgery obstruction is also zero. This implies :

Corollary. If $k+1$ is not a power of 2 and $\nu_2(k) < 5$, then the $(4k+1)$ dimensional Kervaire sphere does not admit any free S^1 action.

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A new bound for the cup-length of zero-cobordant manifolds

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Abstract

In this talk a new upper bound for the $\mathbb{Z}_2$-cup-length of unorientedly zero-cobordant smooth closed connected manifolds will be presented. This bound is similar in its spirit to another upper bound for the $\mathbb{Z}_2$-cup-length, published in J. Korbaš, Topology Appl. 153 (2006), 2976-2986, which works (in particular) for all smooth closed connected manifolds. As an illustration, it will be shown that the new bound leads to interesting estimates, and in infinitely many cases even to exact values, of the $\mathbb{Z}_2$-cup-length of the Grassmann manifolds $SO(n)/SO(k) \times SO(n-k)$ of oriented k-dimensional vector subspaces in Euclidean n-space with n odd. [Since these *homogeneous spaces* are zero-cobordant thanks to an obvious stationary point free *action of the group* $\mathbb{Z}_2$ on them, one can see here one of the many interplays between the two main topics of the present conference.]

To use the upper bound for the $\mathbb{Z}_2$-cup-length of zero-cobordant manifolds *optimally*, one should know the value of a numerical homotopy invariant of smooth closed connected manifolds, apparently not studied up to now, called the characteristic rank (J. Korbaš, Bull. Belgian Math. Soc., to appear): the characteristic rank of a smooth closed connected d-dimensional manifold M is defined to be the largest integer k $(0 \le k \le d)$ such that each element of the cohomology group $H^j(M; \mathbb{Z}_2)$ with $j \le k$ can be expressed as a polynomial in the Stiefel-Whitney classes of M. Some results on the characteristic rank of the Grassmann manifolds $SO(n)/SO(k) \times SO(n-k)$ will also be mentioned.

Simply connected asymmetric manifolds

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Abstract

A closed smooth manifold M is called asymmetric if no compact Lie group can act effectively or equivalently if for all Riemannian metrics the group of self-isometries is trivial. About 40 years ago Borel gave a criterion for aspherical manifolds which guarantees that the manifold is asymmetric. Since many people believe that a manifold "picked at random" is asymmetric there should also be simply connected examples. But this is an open problem for about 30 years. Volker Puppe has shown that there are many 6-dimensional simply connected manifolds which are almost asymmetric in the sense that there are no orientation preserving actions. I showed that all these manifolds are actually asymmetric. The key is some sort of index formula which I will explain.

ON PROJECTIVE BUNDLES OVER SMALL COVERS

SHINTARÔ KUROKI

A *small cover* M is an n-dimensional manifold with locally standard $\mathbb{Z}_2^n$-action whose orbit space is a simple polytope (e.g. the real projective space $\mathbb{R}P(n)$ is a small cover). Small cover can be regarded as a real version of quasitoric manifolds.

Let ξ be an equivariant vector bundle over a small cover M. If ξ decomposes into a Whitney sum of line bundles, then its projectivization $P(\xi)$ becomes a small cover again. We call $P(\xi)$ a *projective bundle over small cover*.

In the paper [1], this bundle was used to find counter examples of cohomological rigidity of small covers, i.e., the cohomology ring over $\mathbb{Z}_2$-coefficient can not distinguish homeomorphism types of small covers. Such counter examples are constructed from projective bundles over $\mathbb{R}P(n)$ by classifying their homeomorphism types. So we are naturally led to study homeomorphism types of projective bundles over every small cover.

In this talk, we characterize such projective bundles over 2-dimensional small covers by using combinatorial argument and introducing a new combinatorial operation $\sharp^{\Delta^{k-1}}$. The main theorem is as follows:

Theorem 1. *Let $P(\xi)$ be a projective bundle over 2-dimensional small cover M^2. Then $P(\xi)$ can be constructed from projective bundles $P(\kappa)$ over the real projective space $\mathbb{R}P^2$ and $P(\zeta)$ over the torus T^2 by using $\sharp^{\Delta^{k-1}}$.*

Here, a combinatorial operation $\sharp^{\Delta^{k-1}}$, where k is a fibre dimension of ξ, corresponds with the fibre sum along the fibre as the geometric operation. Moreover, homeomorphism types of $P(\kappa)$ and $P(\zeta)$ are known by easy computations.

This is a joint work with Zhi Lü.

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SURGERY CLASSIFICATION OF LENS SPACES
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TIBOR MACKO

Let G be a finite cyclic group and let V be a finite-dimensional G-representation which is free away from $0 \in V$. Such a representation gives a free action of G on the sphere SV. The orbit space is a manifold called a *lens space*. The classification of lens spaces up to homotopy equivalence is a standard topic of courses in algebraic topology. The classification up to simple homotopy equivalence and up to homeomorphism goes via the Reidemeister torsion and is a standard topic of courses in algebraic K-theory.

A fake lens space is the orbit space of any free action of a finite cyclic group on a finite dimensional sphere, that means not necessarily one coming from a representation. It turns out that the homotopy and simple homotopy classification of these topological manifolds can be obtained using the same methods as for the standard lens spaces. However, the homeomorphism classification is more complicated, in addition methods of surgery theory are needed. Wall and others in the 1960's treated the case when the order of the group G is $N = 2$ and N is odd, the results are presented in chapter 14 of Wall's book *Surgery on compact manifolds*. However, the case N even remained open, due to lack of knowledge of certain surgery L-groups (which were, however, calculated in the meantime) and due to computational difficulties. In a joint work with Christian Wegner we recently addressed the calculations for the remaining cases (arXiv:0810.1196 and arXiv:0805.0965).

In the talk I will survey the above methods and calculations. Besides surgery theory an important tool is the so-called rho-invariant, which is an invariant of closed odd-dimensional manifolds closely related to the G-signature of even-dimensional manifolds.

The surgery classification is stated in terms of the so-called surgery structure set. It turns out that this set is just π_0 of a certain automorphism space of lens spaces. The above methods can be used to study also the higher homotopy groups of that space. In cases $N = 2$ and N odd this has been done by Madsen and Rothenberg in the 1980's. I will discuss this aspect as well.

It turns out that when the automorphism spaces are studied, yet another tool can be employed, the orthogonal calculus of functors of Weiss. If time permits I will sketch some ideas in this direction.

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Homotopical theory of periodic points of periodic homeomorphisms on closed surfaces

Wacław Marzantowicz & Xuexhi Zhao

Abstract

In this talk we show that the Wecken theorem for periodic points holds for periodic homeomorphisms on closed surfaces, which therefore completes the periodic point theory in such a special case. Using it we derive the set of homotopy minimal periods for such homeomorphisms. Moreover we show that the results hold for homotopically periodic self-maps of closed surfaces. This lets us to re-formulate our results as a statement on properties of elements of finite order in the group of outer automorphisms of the fundamental group of a surface with non-positive Euler characteristic.

SYMMETRY OF TORUS MANIFOLDS

MIKIYA MASUDA

A fundamental result in toric geometry says that there is a bijective correspondence between toric varieties and fans, so all algebrogeometrial information on a toric variety is encoded in the associated fan. Among toric varieties, compact smooth toric varieties, which we call *toric manifolds*, are well studied. If X is a toric manifold, then the group $\mathrm{Aut}(X)$ of automorphisms of X is known to be a (finite dimensional) algebraic group and Demazure introduced a root system for the fan associated with X and proved that it agrees with the root system of the identity component $\mathrm{Aut}^0(X)$ of $\mathrm{Aut}(X)$. He also described the mapping class group $\mathrm{Aut}(X)/\mathrm{Aut}^0(X)$ in terms of the fan associated with X.

A *symplectic toric manifold*, which is a compact symplectic manifold (M, ω) with a Hamiltonian action of a compact torus T where $2\dim T = \dim M$, is a symplectic counterpart to a toric manifold, but the group $\mathrm{Symp}(M, \omega)$ of symplectomorphisms of (M, ω) is infinite dimensional unlike in the toric case. According to Delzant, symplectic toric manifolds are classified by their moment polytopes. In this talk I introduce a root system $R(P)$ for the moment polytope P and show that any irreducible subsystem of $R(P)$ is of type A and that if G is a compact Lie subgroup of $\mathrm{Symp}(M, \omega)$ containing the torus T, then the root system of G is a subsystem of $R(P)$, so that G is of type A. We can also estimate the mapping class group G/G^0 in terms of an automorphism group of P.

A similar story goes in the topological category, to be more precise, for a *torus manifold* M and the group $\mathrm{Diff}(M)$ of diffeomorphisms of M, where a torus manifold M is a compact smooth manifold with a smooth action of a torus T where $2\dim T = \dim M$ and $M^T \neq \emptyset$. The story also goes for a torus manifold M with a stably complex structure J invariant under the action of T and the group $\mathrm{Diff}(M, J)$ of diffeomorphisms of M preserving the J.

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On The Smith Equivalent Representations of Oliver Groups

Masaharu Morimoto and Yan Qi
(Okayama University)

Let G be a finite group. Two real G-modules V and W are called **$\mathfrak{S}_{\mathrm{ht}}$-related** (or **Smith equivalent**) and written $V \sim_{\mathfrak{S}_{\mathrm{ht}}} W$ if there exists a homotopy sphere Σ with smooth G-action such that $\Sigma^G = \{a, b\}$, and $T_a(\Sigma) \cong V$ and $T_b(\Sigma) \cong W$. This Σ is called an **$\mathfrak{S}_{\mathrm{ht}}$-realization** of V and W. Since late 1970's, we have obtained various results which provide $\mathfrak{S}_{\mathrm{ht}}$-realizations of certain nontrivial pairs (V, W). Let $\mathrm{RO}(G, \mathfrak{S}_{\mathrm{ht}})$ denote the **Smith set**

$$\{[V] - [W] \in \mathrm{RO}(G) \mid V \sim_{\mathfrak{S}_{\mathrm{ht}}} W\}$$

and let $\mathrm{RO}(G, \mathfrak{S}_{\mathrm{ht}})_{\mathcal{P}}$ denote the **primary Smith set**

$$\{[V] - [W] \in \mathrm{RO}(G, \mathfrak{S}) \mid \mathrm{res}^G_P V \cong \mathrm{res}^G_P W \quad \forall P \in \mathcal{P}(G)\}$$

where $\mathcal{P}(G)$ is the set of all subgroups of G of prime power order.

Nontrivial $\mathfrak{S}_{\mathrm{ht}}$-equivalent pairs (V, W) were found by Petrie, Cappell-Shaneson, Petrie-Randall, Petrie-Dovermann, Suh, ..., but their pairs satisfy the condition

$$\text{(C)} \qquad \dim V^N = \dim W^N \text{ for any } N \triangleleft G \text{ with prime index.}$$

We would like to find $\mathfrak{S}_{\mathrm{ht}}$-equivalent pairs (V, W) not satisfying Condition (C). Let $N = G^{\mathrm{nil}}$ be the smallest normal subgroup of G such that G/N is nilpotent.

Theorem 1. *Let G be an Oliver group with $|G/N| = 3$ and N has a subquotient group isomorphic to a dihedral group of order $2q$ with an odd prime q. If (V, W) is a $\mathcal{P}$-matched pair of real G-modules such that $V^G = 0$, $V^N \neq 0$, and $W^N = 0$, then $V \oplus U$ and $W \oplus U$ are $\mathfrak{S}_{\mathrm{ht}}$-equivalent for sufficiently large real G-modules U with $U^N = 0$. Moreover in the case $a_G = 2$, $\mathrm{RO}(G, \mathfrak{S}_{\mathrm{ht}})_{\mathcal{P}}$ is isomorphic to $\mathbb{Z}$.*

This enables us to determine the Smith sets of certain groups G, e.g. $G = P\Sigma L(2, 27)$, $SG(864, 2666)$, and $SG(864, 4666)$. Furthermore, we have a generalization of the theorem above and can compute the Smith sets of certain groups G with $|G/N| = 6$, e.g. $G = (D_6 \times D_6 \times D_6) \rtimes C_3$ (a gap Oliver group).

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Isovariant maps from free G-manifolds to representation spheres

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Abstract

This research is joint with F. Ushitaki.

Let M be an n-dimensional connected orientable closed manifold. It is a well-known result, called the Hopf classification theorem, that homotopy classes of continuous maps from M to the n-dimensional sphere S^n are classified by their degree.

We treat an isovariant version of the Hopf classification theorem under the following situation: Let M be a connected, orientable, closed manifold with free G-action, and SW a G-representation sphere (i.e., the unit sphere of unitary representation W of G), and assume

$$\dim M + 1 = \dim SW - \dim SW^{>1},$$

where $SW^{>1}$ denotes the singular set of SW.

A G-equivariant map $f : X \to Y$ is called G-isovariant if f preserves the isotropy groups, i.e., $G_{f(x)} = G_x$ for all $x \in X$. A G-homotopy $F : X \times I \to Y$ is called an isovariant homotopy if F is G-isovariant.

We determine the isovariant homotopy set $[M, SW]_G^{\mathrm{isov}}$ by calculating the equivariant obstruction group. As a result, isovariant maps are classified by the multidegree of isovariant maps ([1], [2]) if G-action on M is orientation-preserving, but not are necessary classified by the multidegree if G-action on M is not orientation-preserving.

We also discuss the case of non-orientable manifolds if time allows.

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ON THE SIGNATURE OF GRASSMANNIANS VIA SCHUBERT CALCULUS

MASAKI NAKAGAWA

Abstract. Let $M = M^{4k}$ be a compact oriented manifold of dimension $4k$. Then the symmetric bilinear form defined by

$$Q_M : H^{2k}(M;\mathbb{R}) \times H^{2k}(M;\mathbb{R}) \longrightarrow \mathbb{R}, \quad (x,y) \longmapsto (x \cup y)\,[M]$$

is non-degenerate by the Poincaré duality theorem. The *signature* $\sigma(M)$ of M is by definition the signature of Q_M. The signature of a manifold whose dimension is not divisible by 4 is defined to be zero. The celebrated Hirzebruch signature theorem asserts that the signature $\sigma(M)$ of a compact oriented manifold $M = M^{4k}$ can be represented as a certain linear combination of Pontrjagin numbers of M. However, the explicit computation of the signature of a manifold M is usually a difficult task, even if its cohomology ring $H^*(M;\mathbb{R})$ and characteristic classes are well known.

In [4], P. Shanahan computed the signature of a complex Grassmannian. He used the Atiyah-Bott Lefschetz formula [1], and the result is given as follows:

Theorem 0.1. *Let $\mathbb{G}(k,n)$ be the complex Grassmannian of k-dimensional linear subspaces of $\mathbb{C}^n$. Then the signature of $\mathbb{G}(k,n)$ is*

$$\sigma(\mathbb{G}(k,n)) = \begin{cases} \binom{[\frac{n}{2}]}{[\frac{k}{2}]} & \text{if } k(n-k) \text{ even}, \\ 0 & \text{if } k(n-k) \text{ odd}. \end{cases}$$

Using the Schubert calculus technique (see [3], [2]), we will give a quite simple proof of the above result. More explicitly, the Schubert classes behave nicely under the Poincaré duality, and the product of two Schubert classes can be described combinatorially. From this, the computation of $\sigma(\mathbb{G}(k,n))$ reduces to a simple enumerative problem, and we obtain the above result.

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Torus actions and comblex cobordism

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This is a joint work with Victor M. Buchstaber and Nigel Ray.

The ideas and methodology of the emerging field of *Toric Topology* can be applied to one of the classical subjects of algebraic topology: finding nice representatives in complex cobordism classes. *Quasitoric manifolds* are the key players; they constitute a sufficiently wide class of stably complex manifolds to represent the whole complex cobordism ring. In other words, every stably complex manifold is cobordant to a manifold with a nicely behaving torus action. More precise statement is as follows.

Theorem 1. *In dimensions > 2, every complex cobordism class contains a quasitoric manifold, necessarily connected, whose stably complex structure is compatible with the action of the torus.*

The first ingredient of the proof is an explicit construction of a series of algebraic smooth toric varieties additively generating the complex cobordism ring; this was completed in an earlier work of Buchstaber-Ray. We proceed by defining a generalisation of the equivariant connected sum operation, which is used to replace the disjoint union of manifolds (representing the sum operation in cobordism) by a single connected quasitoric manifold.

By way of application, we give an explicit construction of a quasitoric representative for every complex cobordism class as the quotient of a free torus action on a real quadratic complete intersection. The latter is a yet another disguise of the *moment-angle manifold*, another familiar object of toric topology. We therefore obtain a systematic description for quasitoric manifolds and their cobordism classes in terms of combinatorial data.

Originally, these considerations take place in non-equivariant complex cobordism; but the combinatorial description allows for calculations with quasitoric representatives in equivariant cobordism also. These theme is currently under development [2].

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CLASSIFICATION OF QUASITORIC MANIFOLDS OF $b_2 = 2$

SEONJEONG PARK

Abstract

A *quasitoric manifold*, introduced by Davis and Januszkiewicz in [1], of dimension $2n$ is a smooth manifold with locally standard half-dimensional torus $(S^1)^n$ action whose orbit space is diffeomorphic to a simple polytope of dimension n as manifolds with corners. A quasitoric manifold can be thought of topological generalization of a compact smooth toric variety, which we call a *toric manifold*. Among varieties, toric manifolds are well studied. In particular, toric manifolds of the second Betti number 2 are already classified as varieties by Kleinschmidt [2]. In this talk, we classify topologically quasitoric manifolds of $b_2 = 2$; they are distinguished by their cohomology rings up to homeomorphism. Especially, we focus on quasitoric manifolds of $b_2 = 2$ which are not homeomorphic to toric manifolds. This work is jointly with Suyoung Choi and Dong Youp Suh.

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MAPS BETWEEN GRASSMANN MANIFOLDS

PARAMESWARAN SANKARAN

Abstract

Let $FG_{n,k}$ denote the Grassmann manifold of k-dimensional F-vector subspaces of F^n where F denotes the field of real numbers or complex numbers. It is an interesting problem to classify maps between two Grassmann manifolds of the same dimension.

Specifically we shall address the question: For what values of k, l, n, m with $k(n-k) = l(m-l)$ is there a non-zero degree map $\mathbb{C}G_{n,k} \longrightarrow \mathbb{C}G_{m,l}$? We shall give an exposition of recent results on this problem. Our results apply to the case of quaternionic Grassmann manifolds as well.

In the case when $F = \mathbb{R}$, the manifold $G_{n,k}$ is not orientable unless n is even. However, the same question can be addressed for maps between oriented Grassmann manifolds $\widetilde{G}_{n,k}$ which is the (universal) double cover of $G_{n,k}$ (when $n > 2$). Although in this case the results were obtained over twelve years ago, we shall recall them (if there is time) for the sake of completeness.

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PARAMETRIZED BORSUK-ULAM PROBLEM FOR PROJECTIVE SPACE BUNDLES

ABSTRACT. Let $\pi : E \to B$ be a fiber bundle with fibers having the mod 2 cohomology algebra of a real or a complex projective space and let $\pi^{'} : E^{'} \to B$ be a vector bundle. Let $\mathbb{Z}_2$ act fiber preserving and freely on both E and $E^{'} - 0$, where 0 stands for the zero section of the bundle $\pi^{'} : E^{'} \to B$. For a fiber preserving $\mathbb{Z}_2$-equivariant map $f : E \to E^{'}$, we estimate the cohomological dimension of the zero set $Z_f = \{x \in E \mid f(x) = 0\}$. As an application, we also estimate the size of the $\mathbb{Z}_2$-coincidence set $A_f = \{x \in E \mid f(x) = f(T(x))\}$ of a fiber preserving map $f : E \to E^{'}$, where $T : E \to E$ is the generator of the $\mathbb{Z}_2$-action and no action on $E^{'}$ is specified.

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SURGERIES ON KNOTS BOUNDING DEFINITE 4-MANIFOLDS

SAŠO STRLE

ABSTRACT

In the study of various properties or structures on a closed 3-manifold Y (eg. fillable contact structures) it is often important to know whether Y bounds a compact definite 4-manifold (of given definiteness). We study this question for surgeries on knots. It turns out that in this case the information can be encoded in a numerical invariant that is a concordance invariant of the surgery knot and has a subadditivity property. We compute this invariant for torus knots.

There are two basic approaches to problems of this sort, both essentially relaying on gauge theory. Though the more numerical approach using correction term invariants in Heegaard-Floer homology of Ozsváth and Szabó (or related invariants in other versions of gauge theoretic invariants) seems more promissing at first it turns out that the computation for torus knots can be carried out using Donaldson's diagonalization theorem.

This is joint work with Brendan Owens.

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PROPERTIES OF BOTT MANIFOLD AND COHOMOLGICAL RIGIDITY

DONG YOUP SUH

A *Bott tower* is a sequence $B_n \to B_{n-1} \to \cdots \to B_1 \to B_0$ where B_0 is a point and B_i is a $\mathbb{C}P^1$-bundle over B_{i-1} for $i = 1, \dots n$. Each B_i is a called the *i-th stage Bott manifold*. One can extend this definition to define a *generalized Bott tower and manifold* to be a sequence of complex space bundles. One of the interesting question in toric topology asks whether two (generalized) Bott manifolds B_n and B'_n are homeomorphic (or diffeomorphic) provided their cohomologies are isomorphic as graded rings. This is called the *cohomological rigidity question* for (generalized) Bott manifolds. In this talk we give some topological and cohomological properties of Bott manifolds, and using these properties we give some positive answers to cohomological rigidity question for Bott manifolds.

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SMITH EQUIVALENT MODULES AND THE WEAK GAP CONDITION

TOSHIO SUMI

In the real representation ring $RO(G)$ of G, we consider the subset $CSm(G)$ consisting of the differences $U - V$ of real G-modules U and V which are *strong Smith equivalent*; i.e., there exists a smooth action of G on a (homotopy) sphere Σ with exactly two fixed points x and y such that

$$T_x(\Sigma) \cong U \oplus W \text{ and } T_y(\Sigma) \cong V \oplus W$$

as real G-modules for some real G-module W and Σ^P is connected for a subgroup P of G of prime power order.

In this talk we discuss what $CSm(G)$ is for an Oliver group G. Here G is an Oliver group if there exists no sequence of subgroups $P \triangleleft H \triangleleft G$ such that P and G/H are groups of prime power order and H/P is cyclic.

Let $\mathcal{P}(G)$ be the set of all subgroups of G of prime power order and let $O^p(G)$ for a prime p denote the smallest normal subgroup G with index a p-power. We denote

$$\bigcap_{P \in \mathcal{P}(G)} \ker(\mathrm{Res}^G_P\colon RO(G) \to RO(P)) \cap \bigcap_p \ker\left(\mathrm{Fix}^{O^p(G)}\colon RO(G) \to RO(G/O^p(G))\right)$$

by $LO(G)$. A group G is called a *gap group* if there exists a G-module W such that $\mathrm{Fix}^{O^p(G)}(W) = 0$ for any prime p and $\dim W^P > 2\dim W^H$ for all pairs (P, H) of subgroups of G with $P \in \mathcal{P}(G)$ and $P < H$.

Theorem. *Let G be an Oliver group. Suppose that for any subgroup K with $[K : O^2(G)] = 2$ it holds that K is a gap group or that all elements x of $K \smallsetminus O^2(G)$ of order 2 such that $C_K(x)$ is not a 2-group are conjugate in G. Then it holds that $LO(G)$ is a subset of $CSm(G)$. Furthermore, if $O^p(G) = G$ for any odd prime p, then it holds that $LO(G) = CSm(G)$.*

We remark that all groups G with $|G| \leq 2000$ satisfy the assumption.

Theorem. *Let G be an Oliver group satisfying that $|G| \leq 2000$ and that $G/(\cap_p O^p(G))$ is an elementary abelian 2-group, and let K be afinite elementary abelian 2-group. Then $CSm(G \times K) = LO(G \times K)$.*

Theorem. *$Sm(PGL(2, q)) = CSm(PGL(2, q)) = LO(PGL(2, q))$.*

TOPOLOGY OF THE GROUP OF HAMILTONIAN SYMPLECTOMORPHISMS OF COMPACT HOMOGENEOUS SPACES

ALEKSY TRALLE

By a, now classical, result of Gromov, the symplectomorphism group

$$\mathrm{Symp}_0(S^2 \times S^2, \omega)$$

for the standard product symplectic form, has the homotopy type of $SO(3) \times SO(3)$. If one changes the product symplectic form $\omega_{S^2} \oplus \omega_{S^2}$ by rescaling one ω_{S^2}, one can get the symplectomorphism group of a different homotopy type. However, the generators of $\pi_* \mathrm{Symp}_0$ are obtained from the map $\pi_* SO(3) \to \pi_* \mathrm{Symp}_0$ induced by the inclusion $SO(3) \to \mathrm{Symp}_0$ in both cases. The following general question is motivated by these results: *is the symplectic action of a compact Lie group G on a closed symplectic manifold (M, ω) visible on the level of $\pi_* \mathrm{Symp}(M, \omega)$?* We prove the following results in this direction.

Theorem 1. *Let K/H be a hermitian symmetric space with a standard K-invariant symplectic form. Then the homomorphism $K \to \mathrm{Ham}(M, \omega)$ induces a surjection on rational cohomology.*

Theorem 2. *Let G/K be a Riemannian symmetric space of non-compact type, and M/K be the Riemannian symmetric space dual to G/K in the sense of Matsushima. Let Γ denote a lattice in G. Consider the Matsushima homorphims $\mu^* : H^*(M/K) \to H^*(B\Gamma) = H^*(\Gamma \setminus G/K)$. Let K/H be any compact hermitian symmetric space. Let $c : B\Gamma \to B\,\mathrm{Ham}(K/H)$ by a map classifying the bundle*

$$K/H \to \Gamma \setminus G/H \to \Gamma \setminus G/K.$$

Then the image of the induced homomorphism

$$c^* : H^*(B\,\mathrm{Ham}(K/H), \mathbb{R}) \to H^*(\Gamma, \mathbb{R})$$

is equal to the image of the Matsushima homomorphism. In other words, all Matsushima classes are Hamiltonian characteristic classes.

These results detect nontrivial elements in $\pi_* \mathrm{Ham}(M, \omega)$, if $M = K/H$ is a hermitian symmetric space. In this way, in particular, we generalize several previous results by Reznikov and Kędra-McDuff. An essential feature of this work is a construction of a new hamiltonian fiber bundle, whose fiber is a compact symplectic homogeneous space. This enables us to use a general machinery of classifying bundles for the group $\mathrm{Ham}(M, \omega)$, and to establish a link between the cohomology of $B\,\mathrm{Ham}(M, \omega)$ and the cohomology of lattices in Lie groups.

This is a joint work with Jarek Kędra.

Quaternionic Toric Varieties

Dariusz M. Wilczyński

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Abstract

We are going to discuss a comparison theorem for the algebraic and topological classifications of quaternionic toric varieties. It turns out that every topological type of an 8-dimensional quaternionic toric variety contains infinitely many distinct algebraic types of such manifolds (joint work with Piotr Runge).

Span of Projective Stiefel Manifolds

Peter Zvengrowski

Abstract

In this talk some of the known results on the span of the real projective Stiefel manifolds $X_{n,r}$ will be summarized, and some recent improvements due to work of P. Sankaran, J. Korbaš, and the author will be presented. A wide variety of techniques have been applied to this problem, including vector bundles, characteristic classes, primary and secondary cohomology operations, K-theory, Cayley-Dickson algebras, normal bordism, etc., leading to sharp and even exact results in many cases. The case $X_{n,2}$ with n odd seems to be the most difficult, and will receive special attention in the talk.